EXAMINATION 07-11-2014

ATOMS AND MOLECULES. 08:30-11:30, K. DUPPENHAL, # QUESTIONS: 6

YOU CAN MAKE USE (IF YOU THINK YOU HAVE TO) OF THE FOLLOWING FORMULA'S:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} g_J$$

1. ELECTRONIC AND FINE STRUCTURE (10 POINTS)

A. Consider Cesium, which is the element on the sixth row of the first column in the periodic table. Give its ground state electron configuration.

B. Give the term for the ground state and the first excited state of the Cesium atom. Briefly explain what spin-orbit coupling is and why the ground state is not split but the first excited state is. The light that is emitted as this split excited state decays to the ground state consist of two components with wavelengths of 852.3 and 894.6 nm. Calculate the size of the spin-orbit coupling constant β [in cm^{-1}].

2. HYPERFINE STRUCTURE, ZEEMAN EFFECT (20 POINTS)

A. The only stable isotope of Cesium, ^{133}Cs, has a nuclear spin of $I = 7/2$. Calculate the g_F factors for the two hyperfine components of the electronic ground state.

B. Make a sketch of the splitting of the ground state of Cesium in low magnetic fields, and label the lines according to their m_F. Then indicate in the same figure the splitting in large magnetic fields, and show the connection between the two regimes. Also indicate in this figure which states are low-field seeking and which states are high-field seeking.

C. Consider an atomic beam of ^{133}Cs in the ground state which is sent on the y-axis through an interaction region with magnets. A cut-through (X-Z plane) of 5 possible magnet arrangements is shown on the next page in Figure 1. Assume that the dimensions of the beam are as indicated by the grey circle, and that the beam before the experiment is nicely collimated, i.e. the transverse velocity is zero throughout the beam. The beam propagates into the paper at some velocity v. Make a sketch of the trajectory of the beam in the Y-Z plane, showing the effect of the magnetic field on the atoms

C$_1$) assuming that they are all are in the $F = 3$ level

C$_2$) assuming that they are all are in the $m_F = +4$ state, and that the atoms enter the local magnetic field adiabatically

D. Supose that you have a pure beam of atoms in the $m_F = +4$ state. Can you think of a way to transform this beam into a pure beam in the $m_F = -4$ state?

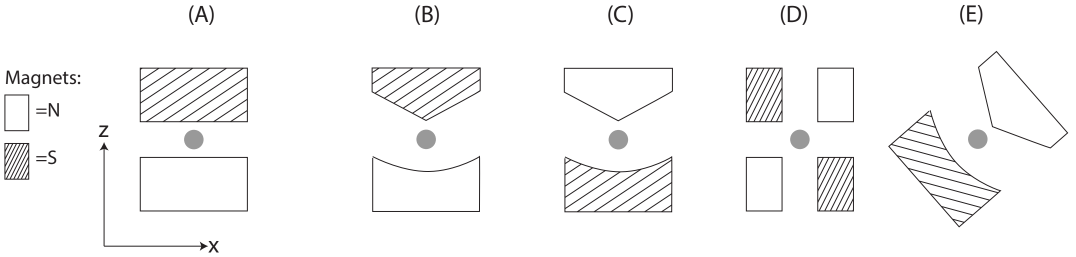

Figure 1. A cut-through of various magnet configurations. An atomic beam of Cesium propagates through these magnets, into the plane of the paper, as indicated by the grey dot.

3. Ramsey spectroscopy and laser cooling (20 points)

A hyperfine transition in the electronic ground state of Cesium is being used for the definition of the second. This is done using Ramsey spectroscopy on laser-cooled atoms.

A. Draw a schematic diagram of a Ramsey setup making use of separated oscillatory fields. Label the essential components, and indicate the relevant quantum numbers (F,m_F) for Cesium atoms in various parts of the setup. Also draw the typical spectrum that is obtained.

B. Explain how this experiment works using pictures of a Bloch sphere with the state vector that describes the state of the Cesium atom. Use these pictures to explain the minima and maxima in the spectrum that is obtained.

C. A typical clock experiment based on the Cesium hyperfine transitions is in the form of an atomic fountain. Draw a schematic overview of such an experiment, illustrating all laser cooling steps required when starting from a hot atomic beam. How is the Ramsey scheme implemented in such a setup?

D. What additional experimental steps would have to be taken to bring the cold Cesium atoms from the previous question to the quantum-degenerate regime?

4. Doppler-free saturation spectroscopy (20 points)

The lasers used in the laser cooling experiments have to be precisely kept at the required wavelength. This is done using Doppler-free spectroscopy. For this question we consider transitions between the hyperfine-splitted states $^2\mathrm{S}_{1/2}$ and $^2\mathrm{P}_{3/2}$ of Cesium.

A. Sketch a typical setup for doppler-free spectroscopy. Explain the operating principle for a simple two-level system.

B. Explain how cross-over peaks can arise, both for a system with two close-lying ground states and for a system with two close-lying excited states.

C. Figure 2 shows the transmission of a probe beam as it is scanned over the $6\ ^2S_{1/2}$ – $6\ ^2P_{3/2}$ transition, resolving the hyperfine states and all possible cross-over peaks that occur for pairs of transitions whose frequency separation falls within the Doppler width. The table gives the relative frequencies of the saturated absorption peaks in MHz. Make a sketch indicating all allowed transitions between the hyperfine substates of the $6\ ^2S_{1/2}$ and $6\ ^2P_{3/2}$ states, and label these transitions with the corresponding letter (A-F and a-f) to explain the doppler-free spectrum of Figure 2.

A	B	C	D	E	F
0	100.7	201.5	226.5	327.2	452.9

a	b	c	d	e	f
0	75.8	151.5	176.5	252.2	353.0

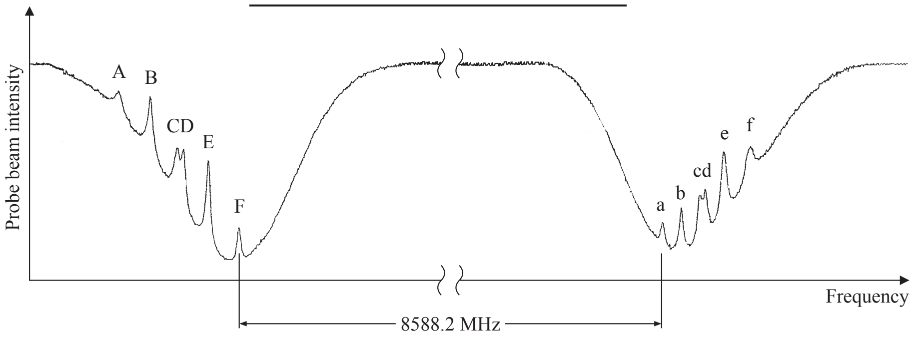

Figure 2. A doppler-free spectrum showing the transmission of a weak probe beam through a sample of Cesium atoms

5. Quantum computation (10 points)

The CNOT gate is an essential ingredient of quantum computation, because it is, when combined with single qubit rotations, the essential building block from which all other quantum operations can be constructed.

A. The CNOT gate has been implemented using trapped ions. Explain how the *control* part of the CNOT gate is realized in such a system.

B. At the heart of the CNOT gate with two trapped ions lies the detection of the phase-shift resulting from a 2π pulse, which is also the essential part of non-destructive photon detection. Show, using the standard formula's that describe the evolution of a two-level system interacting with short excitation pulses, how such a phase-shift can be detected.

6. Diatomic molecules in an external magnetic field (15 points)

A. Give a schematic drawing of the angular momentum coupling for a Hund's case (a) molecule.

B. Give the magnetic moment μ_Ω of these molecular states: a) $^1\Pi$ b) $^3\Sigma$ c) $^2\Pi_{3/2}$ d) $^2\Pi_{1/2}$

C. Without a magnetic field, the internuclear magnetic moment μ_Ω precesses around $\vec{J}$ to form a magnetic moment μ_J. In an external field, $\vec{J}$ precesses around $\vec{B}$. The molecular Zeeman effect for an Hund's case a) molecule strongly depends on J. Make a schematic drawing of the relevant vector quantities to explain this.